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Constrained Hopping Traversability Analysis on Non-Uniform Polygonal Chains

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ABSTRACT We analyze the existence of constrained ballistic hopping paths on a nonuniform polygonal chain. This analysis has practical significance in constrained path planning applications for jumping robots where robot dynamic constraints, uneven surface structure, and non-uniform surface properties are considered. We derive closed-form conditions to satisfy *i)* damage-free robot landing *ii)* non-sliding of the robot *iii)* actuator saturation, and *iv)* intermediate terrain avoidance constraints. Using the closed-form conditions, we propose a traversability algorithm to determine the path existence between two given points on the polygonal chain. Correspondingly, a path generation approach is discussed to generate an optimal constrained hopping path with minimum number of intermediate hops and least take-off speed per hop. Applicability and viability of the proposed algorithms are demonstrated through computer simulations in a realistic terrain scenario with robot jumping motion uncertainties and disturbances.

INDEX TERMS Hopping or jumping robots, motion planning, robot motion, traversability analysis.

I. INTRODUCTION

In general, a hopping robot traverses 3-D surfaces which can be typically represented by a polygonal mesh as shown in Fig. 1. The typical hopping motion between two points takes place in the vertical plane where the 3-D terrain is represented by a polygonal chain given by the intersection of local $X - Z$ plane with the mesh (see the blue solid line in Fig. 1). For a successful hoping, the following major concerns need to be taken into account:

1) robot slipping at the time of take-off and landing which forces restrictions on take-off and landing angles,
2) robot actuator saturation and damage-free landing on a desired waypoint corresponding to bounded take-off and landing speeds respectively, and
3) intermediate terrain avoidance during the hop.

In this work, we are interested in answering the following question: Given the above restrictions, under what conditions there exist feasible single or multi-hop paths between two given points on the non-uniform polygonal chain which satisfy the take-off and landing restrictions, and does not collide with intermediate chain segments during hop.

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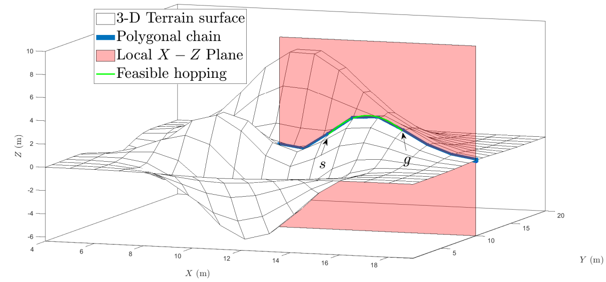


FIGURE 1. A non-uniform polygon chain from a 3-D polygonal surface.

We call this problem as constrained hopping traversability analysis (CHTA) because the emphasize is on determining the path existence conditions. This process should not be confused with a path planning problem where the focus is on the generation of the path. Constrained traversability analysis answers the primary question of solution existence before getting a path which may need sophisticated and computational expensive path-planning algorithms. Note that similar type of problems to the one addressed here are the observability (resp. controllability) question before the observer (resp. controller) design in systems theory. This work considers rapid and agile jumping robots e.g. saltatorial locomotion based robots [1] and proposes traversability analysis which can be used as prerequisite tool for steering in hopping motion planning methods.

A. RELATED WORKS

Traversability analysis is an important prerequisite for constrained motion planning where dynamically constrained robots are traversing constrained regions. For example, in [2]–[5], traversability analysis are carried out to determine the existence of curvature-constrained paths for nonholonomic robots within the polygonal regions. A similar research occurs in case of hopping robots traversing rugged terrains where the robot dynamic constraints and surface properties restrict the hopping motion. With regard to hopping robots, the literature on traversability analysis is very limited. In [6], a global hopping traversability analysis was discussed for a specific case of asteroid exploration. Therein, an asteroid surface was divided in local traversable regions by considering only the terrain slope as metric and their inter-connectivity was obtained by solving the Lambert's orbital boundary value problem for each candidate hop using a numerical shooting method. Subsequently, a graph search approach was used to determine the global traversability.

In contrast to hopping traversability analysis, most of the works about hoping robots focus on robot design [7]–[11] and path planning problems [12]–[16]. Focusing on path planning, a Delaunay triangulation based approach for determining the safe landing sites and hence waypoints amidst point-mass obstacles was proposed in [12]. Therein, A* search algorithm was used for generating feasible hopping paths connecting the determined waypoints. Point-to-point ballistic trajectory generation was discussed in [13], where the take-off and landing velocity vectors of the trajectory were constrained in Coulomb friction cones to ensure the non-slipping and damage-free landing of the point mass hopping robot. This approach was used as steering in a simple probabilistic roadmap (PRM) for generating the feasible path. Another interesting path planning approach for virtual agents with jumping and walking capabilities was presented in [14] where a two level planner with local PRM and a graph search algorithm was proposed to generate a combined path connecting the disconnected or moving regions with jumping motion. However, the works of [12]–[14] only address the single ballistic hop problem to connect two points obtained from the high level path planners. An optimal path planning approach for planetary hopping robot traveling amidst obstacles was discussed in [15] wherein, a heuristic A* search algorithm was used to generate the energy optimal multi-hop paths satisfying maximum height constraint (which relates to maximum landing speed and thus robot damage due to impact). Generation and characterization of multi-hop maximum height constrained point-to-point hopping paths on a plane was discussed in [16]. Therein, all feasible paths were characterized in two categories namely, identical multi-hop and non-identical multi-hop paths, and both types were compared in terms of number of hops and total take-off speed. However, the height constraint is a poor metric of the robot damage as compare to landing speed. Note that for local (point-to-point) planning, typically the jumping motion is represented by the ballistic trajectory [12]–[16].

B. CONTRIBUTIONS

In this work, we focus on determining the existence of a constrained ballistic hopping path between two given points on a known planar polygonal chain with non-uniform speed and angle constraints. The major contribution are as follows:

1) We rigorously define the constrained hopping traversability analysis (CHTA) problem for a ballistic hopping robot on the polygonal chain with non-uniform take-off and landing restrictions (speed and angle constraints), and derive closed-form conditions satisfying these restrictions and intermediate terrain avoidance constraints required for constrained hopping.
2) Using the derived conditions, we develop traversability and path generation (steer) algorithms to determine the connectivity and subsequently, generating an optimal path with minimum number of ballistic hops and least take-off speed.
3) Performance of the proposed CHTA algorithm and its application to path planning are discussed using illustrative examples.

In summary, this work focuses on solving the generalized local traversability analysis problem while satisfying the robot and surface constraints at the local level itself which was not considered in [6]. Moreover, the proposed approach generates optimal paths with multiple constrained hops unlike the work of [13] which checks for a single-hop connectivity between two points.

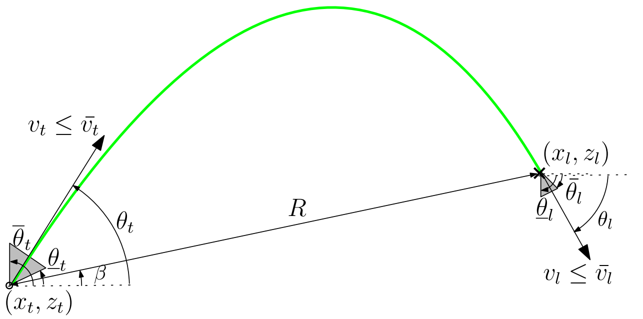
The remainder of the paper is as follows: Section II defines the concerned CHTA problem. Conditions for single hop traversaibility are provided in Section III. Section IV derives the multi-hop traversability conditions and provides the solution to CHTA problems. Simulations with illustrative examples are provided in Section V. Section VI contains concluding remarks.

II. TRAVERSABILITY PROBLEM FOR A HOPPING ROBOT

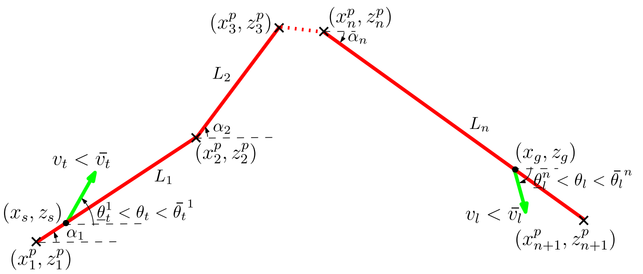
In this section, we first introduce a point-mass hopping robot model and the polygonal chain definition, followed by the hopping restrictions and constraints. At last, we define the concerned traversability problem.

A. MODEL OF A POINT-MASS HOPPING ROBOT

Consider a point-mass robot hopping in a vertical $X-Z$ plane from a take-off point (x_t, z_t) towards a landing point (x_l, z_l) with take-off speed $v_t > 0$ and take-off angle $\theta_t \in (-\pi/2, \pi/2)$ from the $X-$ axis, where without loss of generality, $x_t < x_l$, see Fig. 2a. The robot motion is influenced only by gravity and hence, it follows a ballistic path that can



(a) A 2-D ballistic hopping trajectory satisfying take-off and landing angle constraints (grey cones) and speed constraints.



(b) Polygonal chain terrain surface.

FIGURE 2. The constrained hopping traversability analysis (CHTA) problem schematic.

be described by

$$\ddot{x}(t) = 0 \tag{1}$$

$$\ddot{z}(t) = -g \tag{2}$$

where $(\ddot{x}, \ddot{z})$ are the components of the linear acceleration of the robot and g is the gravitational term, which for simplicity is assumed to be constant throughout the polygonal chain (Fig. 2b). The robot lands at (x_l, z_l) with landing speed $v_l > 0$ and angle $\theta_l \in (-\pi/2, \pi/2)$.

B. POLYGONAL CHAIN

A general polygonal chain on the $X - Z$ axis is defined as follows:

Definition 1: A polygonal chain $P \in \mathbb{R}^2$ in a vertical $X-Z$ plane is a series connection of $n \in \mathbb{Z}^+$ ordered line segments $\{L^i\}_{i=1}^n$ as shown in Fig. 2b. The end positions of each line segment L^i are represented by (x_i^p, z_i^p) and (x_{i+1}^p, z_{i+1}^p), where $x_i^p < x_{i+1}^p$. Each segment L^i makes angle $\alpha_i \in (-\pi/2, \pi/2)$ from X-axis and has length L_i.

C. RESTRICTIONS AND CONSTRAINTS

To ensure non-sliding of the robot at each waypoint, a friction cone based approach [13] is considered. Accordingly, we assume that if the robot's take-off and landing angles are within the friction cone angle limits, the potential sliding of the robot can be neglected. To avoid actuator saturation and robot damage at landing, it is also imposed that the initial and impact speeds, respectively, are upper bounded. The aforementioned constraints can be formally defined as follows:

Definition 2: For a polygonal chain as defined in Definition 1, let $E_f^i := (\underline{\theta}_t^i, \bar{\theta}_t^i, \bar{v}_t^i, \underline{\theta}_l^i, \bar{\theta}_l^i, \bar{v}_l{}^i)$ be referred as line constraints associated to any forward hop in the line segment L^i, $i = 1, 2, \ldots, n$, where $(\underline{\theta}_t^i, \bar{\theta}_t^i)$ (resp. $(\underline{\theta}_l^i, \bar{\theta}_l^i)$) and $\bar{v}_t{}^i$ (resp. $\bar{v}_l{}^i$) represent the take-off (resp. landing) angle limit and maximum take-off (resp. landing) speed. Constraints for the end-point connecting L^i and L^{i+1} are given by the set $(\underline{\theta}_t^{i+1}, \bar{\theta}_t^{i+1}, \bar{v}_t^{i+1}, \underline{\theta}_l^i, \bar{\theta}_l^i, \bar{v}_l{}^i)$. We say that a ballistic hop in forward direction is constrained if all the take-off and landing angles in L^i satisfy the non-sliding constraints, that is, $\underline{\theta}_t^i \leq \theta_t \leq \bar{\theta}_t^i$ and $\underline{\theta}_l^i \leq \theta_l \leq \bar{\theta}_l^i$ and all the take-off and landing speeds are bounded by respective predefined speed bounds, that is, $v_t \leq \bar{v}_t^i$ and $v_l \leq \bar{v}_l^i$. Note that the angular displacement of each cone, that is, $|\underline{\theta}_t^i - \bar{\theta}_t^i|$ and $|\underline{\theta}_l^i - \bar{\theta}_l^i|$ is less than or equal to $\pi/2$, and $v_t^i, v_l^i > 0$.

Similarly, an additional set of constraints (E_r^i) is considered on L^i for the constrained hopping in reverse direction. Note that other undesirable after landing motions (such as bouncing, tumbling, and rolling) are neglected for the traversability problem by considering the properly damped surface and appropriate line constraints [6], [13].

D. PROBLEM DEFINITION

The constrained hopping traversability problem is defined as follows:

Problem 1 (CHTA): Consider a given start $s(x_s, z_s)$ and goal point $g(x_g, z_g)$ on a predefined polygonal chain P as shown in Fig. 2b. Determine, if there exist a constrained hopping path $\Gamma(t) := \{(x, z)|t \rightarrow (x(t), z(t))\} \in \mathbb{R}^2$, for $t \in (t_s, t_g)$ such that

1) (*Constrained hop*): $\Gamma(t_s) = s$ and $\Gamma(t_g) = g$, and s and g are connected with $N \in \mathbb{Z}^+$ intermediate constrained hops as defined in Definition 2.
2) (*Terrain avoidance*): The path Γ does not intersect the polygonal chain except at take-off and landing points.

A second problem that will be addressed in this paper to motivate the CHTA problem is the following:

Problem 2 (Generation): Consider the same setup of Problem 1. In case of existence, generate an admissible path with minimum number of intermediate hops and least take-off speed.

III. CONSTRAINED SINGLE HOP TRAVERSABILITY ON THE POLYGON CHAIN

Useful expressions for the point-to-point ballistic hop between take-off (x_t, z_t) and landing points (x_l, z_l) are provided in Table 1 and their derivations are available in Appendix.

In the sequel, we describe the necessary conditions to ensure single hop traversability between s and g which in this case are the take-off and landing points, respectively, on the polygon chain. For simplicity, in this section we remove the superscript i.

TABLE 1. **Useful expressions for the ballistic hop.**

Properties	Expressions
Velocity components	$\dot{x}(t) = v_t \cos(\theta_t)$ (3a) $\dot{z}(t) = v_t \sin(\theta_t) - gt$ (3b)
Position components	$x(t) = x_t + v_t \cos(\theta_t)t$ (4a) $z(t) = z_t + v_t \sin(\theta_t)t - \frac{gt^2}{2}$ (4b)
Explicit expression of ballistic hop	$z(x) = z_t + \tan(\theta_t)(x - x_t) - \frac{g(x-x_t)^2}{2v_t^2 \cos^2(\theta_t)}$ (5)
Line-of-sight (LOS) distance and angle	$R = \sqrt{(x_t - x_l)^2 + (z_t - z_l)^2}$ (6) $\beta = \tan^{-1} \frac{z_l - z_t}{x_l - x_t}$ (7)
Traveling time	$T = \frac{R \cos\beta}{v_t \cos(\theta_t)}$ (8)
Take-off speed	$v_t = \sqrt{\frac{Rg\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$ (9)
Landing angle and speed	$\theta_l = \tan^{-1}(2\tan(\beta) - \tan(\theta_t))$ (10) $v_l = \sqrt{v_t^2 - 2Rg\sin(\beta)}$ (11)

A. FEASIBLE TAKE-OFF ANGLE LIMIT SATISFYING LANDING ANGLE CONSTRAINTS

Since the landing angle should satisfy

$$\theta_l \leq \bar{\theta}_l$$

then upon substitution from (10), result in

$$\tan^{-1}(2\tan(\beta) - \tan(\theta_t)) \leq \bar{\theta}_l$$
$$\implies \theta_t \geq \tan^{-1}\left(2\tan(\beta) - \tan(\bar{\theta}_l)\right) \quad (12)$$

where β is the angle between the $X-$axis and the line-of-sight (LOS) connecting the take-off and landing points (see Fig. 2a). Similarly, the landing angle should satisfy

$$\theta_l \geq \underline{\theta}_l \implies \theta_t \leq \tan^{-1}\left(2\tan(\beta) - \tan(\underline{\theta}_l)\right) \quad (13)$$

B. AVOIDANCE OF INTERMEDIATE POLYGONAL CHAIN SEGMENTS

The key idea here is to make sure that the ballistic hopping path is always above each intermediate end points of the polygonal chain that leads to intermediate terrain avoidance as shown in Fig. 3. This can be achieved by adding a safety distance $\epsilon > 0$ at all the line segment end points with $x_i^p \in (x_t, x_l)$ and imposing

$$z(x_i^p) \geq z_i^p + \epsilon$$

which upon substitution from (5) and (9) result in

$$\theta_t \geq \tan^{-1}\left(\frac{(z_i^p + \epsilon - z_t)R\cos^2(\beta) - (x_i^p - x_t)^2\sin(\beta)}{(x_i^p - x_t)R\cos^2(\beta) - (x_i^p - x_t)^2\cos(\beta)}\right)$$

Thus, a necessary condition to avoid intersection between the corresponding ballistic path and all the intermediate line segments can be written as

$$\theta_t \geq \max_{x_t < x_i^p < x_l} \tan^{-1}\left(\frac{(z_i^p + \epsilon - z_t)R\cos^2(\beta) - (x_i^p - x_t)^2\sin(\beta)}{(x_i^p - x_t)R\cos^2(\beta) - (x_i^p - x_t)^2\cos(\beta)}\right) \quad (14)$$

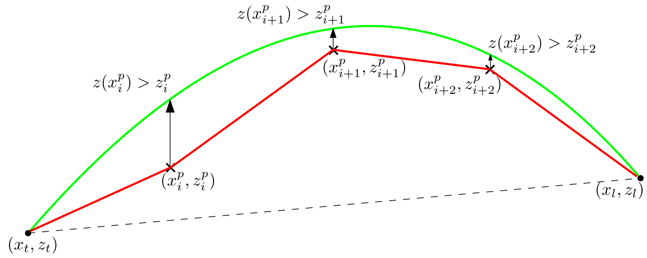


FIGURE 3. **Avoidance of intermediate line segments.**

C. NECESSARY CONDITIONS FOR SPEED CONSTRAINED HOPPING

From the take-off speed constraint

$$v_t \leq \bar{v}_t$$

we obtain upon substitution from (9)

$$\sqrt{\frac{Rg\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}} \leq \bar{v}_t$$
$$\implies \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\left\{\frac{Rg\cos^2(\beta) + \bar{v}_t^2\sin(\beta)}{\bar{v}_t^2}\right\} \leq \theta_t$$
$$\leq \frac{\pi}{2} + \frac{\beta}{2} - \frac{1}{2}\sin^{-1}\left\{\frac{Rg\cos^2(\beta) + \bar{v}_t^2\sin(\beta)}{\bar{v}_t^2}\right\} \quad (15)$$

subjected to

$$\frac{Rg\cos^2(\beta) + \bar{v}_t^2\sin(\beta)}{\bar{v}_t^2} \leq 1$$
$$\implies R \leq \frac{\bar{v}_t^2(1 - \sin(\beta))}{g\cos^2(\beta)} \quad (16)$$

Similarly, using (11), the condition satisfying the landing speed constraint can be derived as

$$v_l \leq \bar{v}_l$$
$$\implies \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\left\{\frac{Rg\cos^2(\beta)}{\bar{v}_l^2 + 2Rg\sin(\beta)} + \sin(\beta)\right\} \leq \theta_t$$
$$\leq \frac{\pi}{2} + \frac{\beta}{2} - \frac{1}{2}\sin^{-1}\left\{\frac{Rg\cos^2(\beta)}{\bar{v}_l^2 + 2Rg\sin(\beta)} + \sin(\beta)\right\} \quad (17)$$

subjected to

$$\frac{Rg\cos^2(\beta)}{\bar{v}_l^2 + 2Rg\sin(\beta)} + \sin(\beta) \leq 1$$
$$\implies R \leq \frac{\bar{v}_l^2}{g(1 - \sin(\beta))} \quad (18)$$

Note that (16) and (18) corresponds to the achievable LOS distances in a single hop for given $\bar{v}_t$ and $\bar{v}_l$, respectively, and hence, the achievable maximum LOS distance satisfying both speed constraints in a single hop can be written as

$$R_{max} = \min\left\{\frac{\bar{v}_t^2(1 - \sin(\beta))}{g\cos^2(\beta)}, \frac{\bar{v}_l^2}{g(1 - \sin(\beta))}\right\} \quad (19)$$

which from (9) and (11), corresponds to a take-off angle

$$\theta_{Rmax} = \frac{\pi}{4} + \frac{\beta}{2} \quad (20)$$

D. PATH EXISTENCE ANALYSIS

Solution to the Problem 1 in the single-hop case is given in Proposition 1 followed by the optimality analysis in Proposition 2.

Proposition 1: For the particular case of imposing the additional constraint of only a single hop, the CHTA Problem 1 is solved as follows: There exist constrained single-hop paths connecting s and g on P provided that the take-off angle satisfies $\theta_t \in \Theta$, with

$$\Theta := \left\{\theta_t | (\theta_t \geq \theta_t^-) \cap (\theta_t \leq \theta_t^+), \forall \theta_t \in (-\pi/2, \pi/2)\right\} \quad (21)$$

where, $\theta_t^- = \max$, as shown at the bottom of the next page, subjected to $R \leq R_{max}$, where R, β and R_{max} given by (6), (7) *and* (19), *represent the LOS distance, LOS angle, and maximum achievable LOS distance, respectively. If $\Theta = \emptyset$ or $R > R_{max}$, then there does not exist any feasible single hop path.*

Proof: The determination of Θ follows directly from (12), (13), (14), (15), (17), and (19), and Definition 2. □

Proposition 2: For the single-hop case discussed in Proposition 1 and a non-empty set Θ as defined by (21), *an optimal constrained single-hop path in terms of least take-off speed for fixed R and β corresponds to an optimal take-off angle $\theta_t^\star$ defined as*

$$\theta_t^\star = \begin{cases} \max(\theta_t^-, \theta_{Rmax}), & \text{for } \theta_{Rmax} \leq \theta_t^+ \\ \theta_t^+, & \text{for } \theta_{Rmax} > \theta_t^+ \end{cases} \quad (22)$$

where, $\theta_{Rmax} = \pi/4 + \beta/2$.

Proof: By taking the partial derivative of (9) with respect to θ_t, equating to zero, and proceeding similarly with (11), it can be concluded that for fixed R and β, the minimum value of take-off speed is obtained at the take-off angle $\theta_t = \pi/4 + \beta/2$, which from (20), is also the required angle for achieving the maximum LOS distance. Hence, if $\theta_t^\star \in \Theta$, then $\theta_t^\star$ corresponds to the optimal constrained single-hop path, otherwise $\theta_t = \theta_t^-$ (resp. θ_t^+) provides the optimal feasible path in $\theta_t^\star \leq \theta_t^+$ (resp. $\theta_t^\star > \theta_t^+$) case. □

Remark 1: From Propositions 1 and 2, it can be concluded that if two points on the polygonal chain are connected by a constrained single hop, then there always exists a single-hop path corresponding to $\theta_t^\star$ with lower take-off speed. Also, the landing speed as given by (11) depends directly on take-off speed for fixed R and β, and hence it has minimum value at $\theta_t^\star$.

IV. CONSTRAINED MULTI-HOP TRAVERSABILITY

In this section, we provide CHTA for $R > R_{max}$ scenarios. At first, we derive the necessary CHTA conditions for the case with s and g on the same line segment (L_i). Next, we present CHTA for any (s, g) pair on the polygonal chain.

A. CHTA ON A LINE SEGMENT

To address $R > R_{max}$ limitation on a line segment, (see Fig. 4) a multi-hop ($N-$hop) traversability approach is needed.

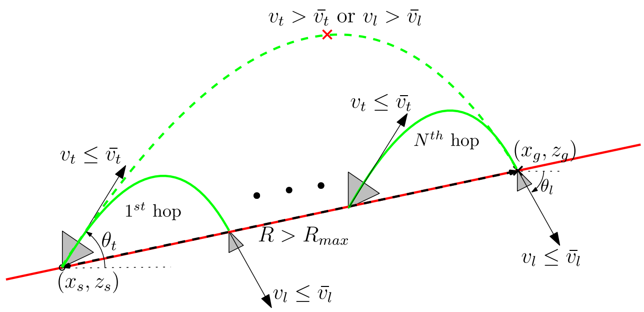


FIGURE 4. A constrained $N-$hop path on a line segment for $R > R_{max}$.

The necessary and sufficient condition for solving the Problem 1 on the same line segment without considering the speed bounds is given in Proposition 3 as follows:

Proposition 3: For the CHTA problem 1 *with s and g on the same line segments of P, and $R > R_{max}$, there exist feasible multi-hop angle constrained paths connecting the given s and g on P provided that the take-off angle in each hop satisfies $\theta_t \in \Theta$ where Θ is defined as*

$$\Theta := [\theta_t^-, \theta_t^+] \quad (23)$$

with the limits

$$\theta_t^- = \max\{\underline{\theta}_t, \tan^{-1}\left(2\tan(\beta) - \tan(\bar{\theta}_l)\right)\} \quad (24)$$

$$\theta_t^+ = \min\{\bar{\theta}_t, \tan^{-1}\left(2\tan(\beta) - \tan(\underline{\theta}_l)\right)\} \quad (25)$$

Proof: From the take-off angle constraint as defined in Definition 2 and the conditions satisfying landing angle constraints as given by (12) and (13), it follows that the feasible set of take off angle satisfying line angle constraints can be given as

$$\theta_t^- \leq \theta_t \leq \theta_t^+$$

□

Now, to satisfy the line speed constraints with angle constraints, the main idea is to divide the given LOS distance in N segments such that the robot travels each segment in a single hop satisfying the maximum possible LOS distance under the line constraints. Proposition 4 provides the minimum number of hops required for feasible constrained hopping on a line segment.

Proposition 4: For a CHTA problem 1 *with additional constraints of Proposition 3 and a non-empty set Θ as given by* (23), *there exist feasible constrained multi-hop paths connecting the given s and g on P, comprised of a minimum number of hops $N \in \mathbb{Z}^+$ given by*

$$N = \max\{N_t, N_l\} \quad (26)$$

where

$$N_t = \left\lceil \frac{Rg\cos^2(\beta)}{2\bar{v}_t^2\cos(\theta_t)\sin(\theta_t - \beta)} \right\rceil \quad \textit{and} \quad (27)$$

$$N_l = \left\lceil \frac{Rg(\cos^2(\beta) - 4\sin(\beta)\cos(\theta_t)\sin(\theta_t - \beta))}{2\bar{v}_l^2\cos(\theta_t)\sin(\theta_t - \beta)} \right\rceil \quad (28)$$

and the Ceiling function $\lceil . \rceil$ *computes the least following integer for the given value.*

Proof: For a fixed $\theta_t \in \Theta$ with Θ given by (23), the covered LOS distances corresponding to line speed bounds ($\bar{v}_t$ and $\bar{v}_l$) can be obtained from the robot velocity components (given by (3a) and (3b)) as

$$R_t = \frac{2\bar{v}_t^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)}{g\cos^2(\beta)} \tag{29}$$

$$R_l = \frac{2\bar{v}_l^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)}{g(\cos^2(\beta)-4\sin(\beta)\cos(\theta_t)\sin(\theta_t-\beta))} \tag{30}$$

respectively. Hence, the number of required feasible segments or hops $N_t \in \mathbb{Z}^+$ that the robot will travel to satisfy the take-off speed constraint can be computed by generalizing (29) as

$$\frac{R}{N_t} = R_t = \frac{2\bar{v}_t^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)}{g\cos^2(\beta)}$$

which implies (27). Similarly, the minimum number of hops required to satisfy the landing constraints can be obtained from (30) as

$$\frac{R}{N_l} = \frac{2\bar{v}_l^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)}{g(\cos^2(\beta)-4\sin(\beta)\cos(\theta_t)\sin(\theta_t-\beta))}$$

and thus (28). Hence, the minimum number of hops required to satisfy the line constraints satisfies (26), for a chosen value of $\theta_t \in \Theta$ given by (23). □

Remark 2: Using Proposition 2 and Remark 1, the optimal take-off angle $\theta_t^\star$ *for the multi-hop scenario can be obtained by substituting* θ_t^- *and* θ_t^+ *from* (24) *and* (25)*, respectively, in* (22) *which corresponds to least take-off speed and maximum achievable distance on LOS satisfying given speed bounds in a single hop. Also, from* (26)*, a chosen* $\theta_t \in \Theta$ *corresponds to a unique value of* N*. Hence, it can be concluded that* $\theta_t^\star$ *provides the minimum number of constrained hops with least take-off speed (also the least landing speed) to connect a given* (s, g) *pair in the multi-hop scenario.*

Using Propositions 3 and 4, and Remark 2, pseudo-code for computing a minimum number of hops for satisfying line constraints are provided in Algorithm 1.

Algorithm 1 MultihopLine: Computation of Minimum Number of Hops N Satisfying Line Constraints on a Line Segment

INPUT: LOS distance R, LOS angle β, line constraints E
OUTPUT: Minimum number of hops N

Compute feasible take off angle set Θ from (23)
if $\Theta \neq \emptyset$ **then**
  $\theta_t \leftarrow \theta_t^\star$ from Remark 2
  $N_t \leftarrow \left\lceil \frac{Rg\cos^2(\beta)}{2\bar{v}_t^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)} \right\rceil$
  $N_l \leftarrow \left\lceil \frac{Rg(\cos^2(\beta)-4\sin(\beta)\cos(\theta_t)\sin(\theta_t-\beta))}{2\bar{v}_l^{\,2}\cos(\theta_t)\sin(\theta_t-\beta)} \right\rceil$
  $N \leftarrow \max(N_t, N_l)$ ▷ Minimum number of hops
else
  $N = 0, \theta_t = \emptyset$
end if

B. CONSTRAINED MULTI-HOP TRAVERSABILITY ON THE POLYGONAL CHAIN

In this section, we provide a generalized traversability analysis for the CHTA problem. We combine the CHTA analysis of Section III and IV-A to build a connectivity graph and use a graph search algorithm to determine the traversability.

1) GRAPH CONSTRUCTION

Consider a topological graph $G = \{V, E\}$ with V nodes and E edges. For a given CHTA problem, we first discretize each line segment with k equidistant discrete points per segment. Subsequently, all the end points and discrete points of the polygonal chain segments together with source s and goal g are stored as nodes in G, and their inter-connectivity in terms of minimum number of required hop are stored as edges in E. The connectivity between any two graph nodes is computed as follows:

$$\theta_t^- = \max\left\{\begin{array}{c} \underline{\theta}_t,\ \tan^{-1}\left(2\tan(\beta)-\tan(\bar{\theta}_l)\right), \\ \max\limits_{x_t<x_i^p<x_l} \tan^{-1}\left(\dfrac{(z_i^p+\epsilon-z_t)R\cos^2(\beta)-(x_i^p-x_t)^2\sin(\beta)}{(x_i^p-x_t)R\cos^2(\beta)-(x_i^p-x_t)^2\cos(\beta)}\right), \\ \dfrac{\beta}{2}+\dfrac{1}{2}\sin^{-1}\left\{\dfrac{Rg\cos^2(\beta)}{\bar{v}_t^2}+\sin(\beta)\right\}, \\ \dfrac{\beta}{2}+\dfrac{1}{2}\sin^{-1}\left\{\dfrac{Rg\cos^2(\beta)}{\bar{v}_t^2+2Rg\sin(\beta)}+\sin(\beta)\right\} \end{array}\right\}$$

$$\theta_t^+ = \min\left\{\begin{array}{c} \bar{\theta}_t,\ \tan^{-1}\left(2\tan(\beta)-\tan(\underline{\theta}_l)\right), \\ \dfrac{\pi}{2}+\dfrac{\beta}{2}-\dfrac{1}{2}\sin^{-1}\left\{\dfrac{Rg\cos^2(\beta)}{\bar{v}_t^2}+\sin(\beta)\right\}, \\ \dfrac{\pi}{2}+\dfrac{\beta}{2}-\dfrac{1}{2}\sin^{-1}\left\{\dfrac{Rg\cos^2(\beta)}{\bar{v}_l^2+2Rg\sin(\beta)}+\sin(\beta)\right\} \end{array}\right\}$$

- For the two nodes which are not on the same line segment L_i and the LOS distance between those pair of points is less than R_{max}, take-off angle solution set is obtained from (21). In case of non-empty solution set, store $N = 1$ as the edge value, otherwise, for the empty Θ or $R > R_{max}$, set $N = 0$, that is, no direct connectivity.
- Connectivity between two nodes on the same line segment, is given from Algorithm 1 where required minimum number of hops is computed from (26) in case of connectivity, otherwise, $N = 0$ is stored.

The aforementioned procedure to compute the feasible number of hops between any two points is given in Algorithm 2.

Algorithm 2 NComp: Computation of Minimum Number of Hops

INPUT: (x_t, z_t), (x_l, z_l), Polygonal chain P coordinates and respective all line angle and speed bounds E,

OUTPUT: Minimum number of hops N

LOS distance $R \leftarrow \sqrt{(x_t - x_l)^2 + (z_t - z_l)^2}$
LOS angle $\beta = \tan^{-1} \frac{z_l - z_t}{x_l - x_t}$
$R_{max} \leftarrow \min \left\{ \frac{\bar{v}_l^2 (1-\sin(\beta))}{g \cos^2(\beta)}, \frac{\bar{v}_l^2}{g(1-\sin(\beta))} \right\}$
if $R < R_{max}$ **then**
  Compute take off angle limit Θ from (21)
  if $\Theta \neq \emptyset$ **then**
    $\theta_t \leftarrow \theta_t^\star$ from (22)
    Set $N = 1$
  else
    Set $N = 0$, $\theta_t = \emptyset$
  end if
else
  if (x_t, z_t) *and* (x_l, z_l) *lie on the same* L_i **then**
    $[N, \theta_t] = \text{MultihopLine}(R, \beta, E)$
  else
    $N = 0$, $\theta_t = \emptyset$
  end if
end if

2) PATH EXISTENCE AND SOLUTION TO PROBLEM 1

To determine the connectivity between the given s and g, we use Dijkstra's graph search algorithm on G which provides a path node sequence N between s and g in case of path existence, otherwise report failure. This traversability analysis (summarized in Algorithm 3) answers the CHTA Problem 1 and provides the input node sequence for solving Problem 2.

C. SOLUTION TO PROBLEM 2

To solve Problem 2, feasible path segments are generated between the consecutive nodes of the N as follows:

1) For generating a single hop between a consecutive node pair, the optimal take-off angle and take-off speed are computed from (22) and (9), respectively.

Algorithm 3 CHTA: Traversability Analysis on the Polygon Chain

INPUT: Polygonal chain P coordinates and all constraints E, $s(x_s, z_s)$, $g(x_g, z_g)$

OUTPUT: $N \neq \emptyset$ (path exist) or $N = \emptyset$ (no solution)

$(x_d, y_d) = discrete(P, k)$ ▷ Discretize each line segment with step size of k and store
$V \leftarrow \left\{ \{(x_i, y_i)\}_{i=1}^{n+1}, (x_d, y_d), (x_s, z_s), (x_g, z_g) \right\}$ ▷ Store all end points and discrete points of the chain with s and g as nodes in V
for all node pairs in V **do** ▷ Compute edge values for G
  $[e, \theta] = \text{NComp}(Node1, Node2, P, E)$
  $\{E\} \leftarrow e$ ▷ Store required minimum number of hops
end for
$G \leftarrow \{V, E\}$ ▷ Store node and edges in G
$N = \text{Dijkstras}(G, s, g)$ ▷ Shortest node sequence in G connecting s and g

2) In case of $N-$hop path segment, the feasible take-off angle θ_t is computed from Algorithm 1 which is used for all N hops. Now, for the first $N - 1$ hops, the maximum feasible LOS distance and take-off speed per hop are computed as

$$R_m = \min\{R_t, R_l\}$$
$$v_t = \sqrt{\frac{R_m g \cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$$

where R_t and R_l are given by (16) and (18), respectively. For the last (N^{th}) hop of remaining LOS distance $R_r = R - (N - 1)R_m$ the take-off speed is computed from

$$v_t = \sqrt{\frac{R_r g \cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$$

All the computed take-off angle and speed sets are used to generate the feasible path connecting the solution node sequence N using (4a) and (4b). Steps for generating a feasible path from N is given in Algorithm 4.

Remark 3: The generated path (robot motion) as governed by (1) *and* (2)*, occurs in the X-Z plane which is aligned with each piece of the polygonal chain. This 2-D path can be converted to a 3-D path by including the $Y-$axis component with respective angle ψ in the $X-Y$ plane. One such example is shown in Section V.B.*

D. PROPERTIES OF THE PROPOSED CHTA SOLUTIONS

This section addresses the completeness, optimality, and time complexity of the proposed approach.

Property 1 (Resolution Completeness): Consider the proposed approach described by Algorithms 1-3 for which the Polygonal chain (defined in Definition 1) data, all line constraints, and a pair of source and goal positions (s, g) are given a priori. Then there exist a sufficiently large number of discrete points k per segment, such that the proposed

Algorithm 4 PathGeneration: Feasible Path Generation From the Feasible Node Sequence

INPUT: Feasible node sequence **N** $\{(x_n, z_n)\}_{n=1}^{|N|}$, number of intermediate hops N between two consecutive nodes of N from G, polygonal chain P, line constraints E
OUTPUT: Optimal path Γ

$\Gamma \leftarrow \emptyset$
for $n = 1$ to $|N| - 1$ **do**
  $[N, \theta_t] = NComp((x_n, z_n), (x_{n+1}, z_{n+1}), P, E)$
  if $N = 1$ **then**
    $v_t = \sqrt{\frac{Rg\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$
    Generate single hop path segment Γ_n from (5) using computed (θ_t, v_t) and given (x_n, y_n).
  else if $N > 1$ **then**
    for $i = 1$ to $N - 1$ **do**
      $R_t \leftarrow \frac{2\bar{v}_t^2 \cos(\theta_t)\sin(\theta_t-\beta)}{g\cos^2(\beta)}$
      $R_l \leftarrow \frac{2\bar{v}_l^2 \cos(\theta_t)\sin(\theta_t-\beta)}{g(\cos^2(\beta) - 4\sin(\beta)\cos(\theta_t)\sin(\theta_t-\beta))}$
      $R_m \leftarrow \min\{R_t, R_l\}$
      $v_t = \sqrt{\frac{R_m g\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$
      Generate $N - 1$ subsequent hops from (5) using computed (θ_t, v_t) starting from (x_n, y_n) which gives path segment $\Gamma_{N-1} = \bigcup_{i=1}^{N-1} \Gamma_i$
    end for
    $R_r \leftarrow R - (N-1)R_m$;
    $v_t = \sqrt{\frac{R_r g\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}}$
    Generate the last hop Γ_r of computed (v_t, θ_t) from (5) from point $(x_t + R_m(N-1)\cos(\beta), R_m(N-1)\sin(\beta))$
    $\Gamma_n \leftarrow \Gamma_{N-1} \bigcup \Gamma_r$
  end if
  Path $\Gamma \leftarrow \Gamma \bigcup \Gamma_n$
end for

CHTA given by Algorithm 3 will determine the traversability and provide the feasible solution node sequence if it exists, otherwise reports failure.

Proof: Equations (21) and (23) answer the path existence between two points on P. Thus, for a chosen $\theta_t \in \Theta$, if k is large enough, there is a sufficiently adjacent nodes such that s and g can be connected to each other directly or through graph nodes under the line constraints. From this, completeness follows because Dijkstra's algorithm is complete [17]. Note that the line constraints and the chain data are given a priori, meaning that k is a design parameter. Thus, the proposed approach can be considered as resolution complete [18]. □

Property 2 (Optimality): In case of path existence from Algorithm 3 for a chosen discrete step size k, Algorithm 4 provides an optimal constrained path connecting s and g with least number of hops and least take-off speed among all feasible θ_t.

TABLE 2. Computational complexity for the proposed approach.

Operation	Time complexity
Algorithm 1 and 2:	Constant $\mathcal{O}(1)$
Algorithm 3: Graph construction Dijkstras's search Remaining	Quadratic time $\mathcal{O}(N^2)$ $\mathcal{O}(N^2)$, for $\lvert N\rvert$ number of nodes $\mathcal{O}(\lvert E\rvert + \lvert N\rvert\log\lvert N\rvert)$ for $\lvert E\rvert$ graph edges and $\lvert N\rvert$ nodes $\mathcal{O}(1)$
Algorithm 4: Single hop path generation Multi-hop path generation	Linear time $\mathcal{O}(2N)$ $\mathcal{O}(N)$, for $\lvert N\rvert$ hops $\mathcal{O}(2N)$, for $\lvert N\rvert$ hops and 2 hops between each subsequent pairs

Proof: As discussed in the Remarks 1 and 2, the proposed approach generates minimum number hops between any two points with least take-off speed, and from the fact that the Dijkstra's search provides the shortest cost path, it can be concluded that Algorithms 3 and 4 provide the optimal $\theta_t^\star$ and generates an optimal path connecting s and g. □

1) COMPUTATIONAL COMPLEXITY

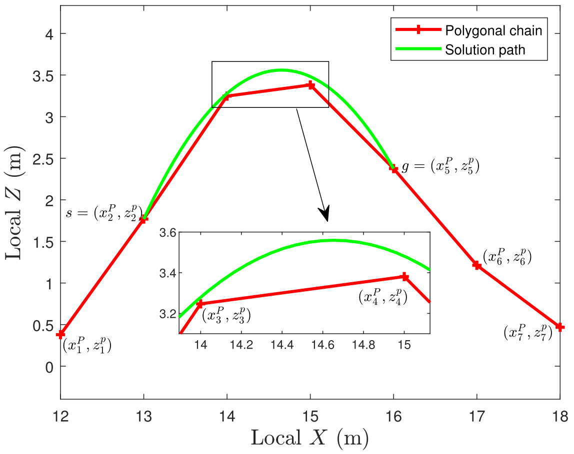
Table 2 summarizes the time complexity for all four algorithms in worst case. It can be seen that the CHTA (Algorithm 3) is the most expensive (quadratic time complexity) among all four algorithms. Notice however that the use of CHTA reduces the time complexity of the path planning, as the traversability and minimum number of hops are determined in advanced and planning is carried out only on the feasible polygonal chains.

V. SIMULATION RESULTS

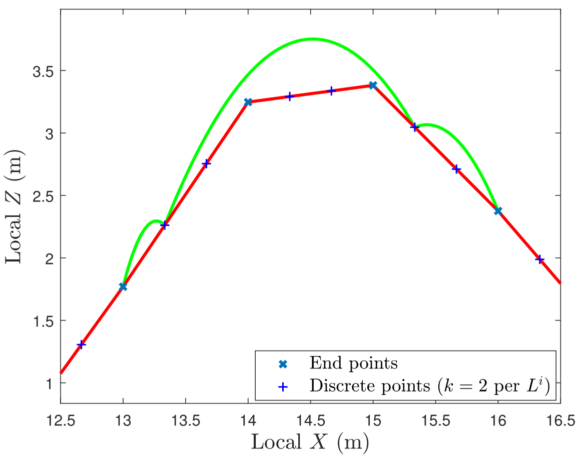
In this section, we first consider a simple scenario to emphasize the basic aspects (applicability and resolution completeness) of the proposed approach. Next, we test the proposed solutions (Algorithms 3 and 4) in a realistic scenario with uncertainties to demonstrate the practical significance of our results.

A. CHTA APPLICATION TO A SIMPLE SCENARIO

We apply the proposed algorithms to determine traversability and generate local paths in a simple 3-D terrain scenario represented by polygon mesh as shown in Fig. 1. At first, a local vertical $X - Z$ plane (Shaded red plane in Fig. 1) passing through a given s and g is defined. The intersection of the local plane with the 3-D surface gives a polygonal chain (solid red line in Fig. 5a) which is an input for the CHTA problems. Line constraints for the polygon chain are given in Table 3. With this, the CHTA Algorithm 3 determines the path existence and provides the shortest node sequence between the (s, g) pair, which in this example corresponds to the nodes (2, 5). Using the graph G data, we determine that only one hop is required to connect node 2 and 5. The corresponding single-hop constrained path is generated from Algorithm 4 and plotted by the solid green line in Fig. 5a. The generated path does not intersect with the intermediate line segments and hence, ensures intermediate terrain avoidance. The simulations were performed on a 1.90 GHz Intel(R) Core(TM) i3-3227U CPU



(a) Single-hop constrained path for $(\bar{v_t}, \bar{v_l}) = (6, 5)$ m/s.



(b) Mult-hop constrained path with $k = 2$.

FIGURE 5. **CHTA and path planning in the simple scenario.**

with 9 GB RAM and the algorithms were implemented in MATLAB. Algorithm 3 (CHTA) determines the traversability in 0.15 s and the Algorithm 4 generates the feasible path in 0.03 s, which highlights the low computation cost of the proposed approach.

1) RESOLUTION COMPLETENESS AND SELECTION OF *K*

To test the resolution completeness, we reduce the take-off speed bound and landing speed bound of L^2 and L^4 to $\bar{v_t} = 6$ and $\bar{v_l} = 5$ m/s, respectively, which leads to no direct connection between s and g due to speed bound violation. Moreover, we first apply Algorithm 3, but considering only the end-points of p (no discretization on the line segments). It turns out that the constraints for segment L^3 leads to an empty set Θ for forward hopping and hence, no direct connection between end-points 3 and 4. Next, we increase k and summarize the outcomes of Algorithm 3 in Table 4, which shows the path existence at $k = 2$. The corresponding feasible 3-hop path is shown in Fig. 5b by the solid green line. Note that, after successful path existence determination, the minimum number of required hops N decreases with increasing k until it reaches the lowest possible value which in this case is $N = 3$.

In summary, the parameter k is related to R_{max} given by (19). Hence, a possible approach to select the value of k is to choose a value such that the R_{max} is less than or equal to the LOS distance between two subsequent nodes on the polygonal chain and then check for path existence using Algorithm 3. In case of non-existence, traversability should be rechecked for increasing values of k until the solution exist or k reaches a maximum permissible value.

TABLE 3. **Simulation data for the polygonal chain.**

Line segments	Line constraints $(\underline{\theta_t}, \bar{\theta_t}, \bar{v_t}, \underline{\theta_l}, \bar{\theta_l}, \bar{v_l})$ (angles in deg., speeds in m/s.)
L^1	$E_f = (55, 85, 2.5, -85, -55, 2.5)$
	$E_r = (55, 130, 2.5, -110, -70, 2.5)$
L^2	$E_f = (55, 90, 7/6, -85, -45, 2.5)$
	$E_r = (5, 85, 2.5, -85, -45, 2.5)$
L^3	$E_f = (90, 110, 2.5, -85, -10, 2.5)$
	$E_r = (80, 170, 2.5, -135, -5, 2.5)$
L^4	$E_f = (-15, 50, 2.5, -90, -60, 6/5)$
	$E_r = (45, 110, 2.5, -45, 40, 2.5)$
L^5	$E_f = (10, 55, 2.5, -120, -55, 2.5)$
	$E_r = (50, 135, 2.5, -120, -55, 2.5)$
L^6	$E_f = (45, 85, 2.5, -130, -45, 2.5)$
	$E_r = (45, 135, 2.5, -120, -55, 2.5)$

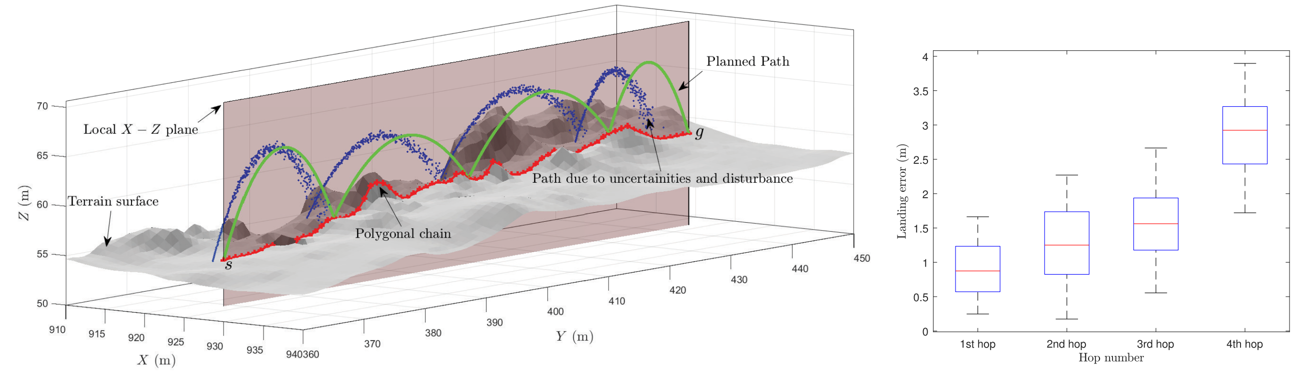
TABLE 4. **Resolution completeness: Effect of varying *k* on CHTA analysis for the simple scenario.**

Discrete points k per line segment	CHTA outcome (number of hops N)
0	Failure, $(N = 0)$
1	Failure, $(N = 0)$
2	Success, $(N = 3)$
3	Success, $(N = 3)$

B. A COMPLEX SCENARIO WITH UNCERTAINTIES AND UNDESIRABLE MOTIONS

We consider a realistic terrain of Mars (data obtained from Mars Trek [19]) and plotted it as a polygonal mesh using MATLAB as shown in Fig. 6a. At first, we generate the 2-D local vertical plane connecting the $s = (923, 369, 54.61)$ m and $g = (923, 445, 59.24)$ m with intermediate end points as discussed in Section V-A which is shown by the transparent plane in Fig. 6a. In sequel, we define forward take-off and landing angle constraints of $\underline{\theta_t} = \alpha_i + \pi/3, \bar{\theta_t} = \alpha_i + \pi/2, \underline{\theta_l} = \alpha_i - \pi/3, \bar{\theta_l} = \alpha_i - \pi/2$, and $\bar{v_t} = \bar{v_l} = 10$ m/s, and consider $g = 3.711$ m/s^2. The proposed CHTA Algorithm 3 determines the traversability with minimum 4 hops and the corresponding solution path is generated using the Algorithm 4 which is plotted by the solid green line in Fig. 6a.

Now, we demonstrate the effect of uncertainties and undesirable robot motions on the proposed approach. To capture the effects due to environment and modeling uncertainties and disturbances, we added white Gaussian noises of zero mean and 0.1 variance in respective (x, y, z) equations of robot motion. Robot drifting from the desired landing point



(a) Optimal planned path and uncertain path in the realistic terrain.

(b) Landing error propogation due to uncertainitites and undesirable robot motions.

FIGURE 6. Effect of the uncertainties and undesirable robot motions on the proposed approach in the Mars terrain scenario.

due to undesirable robot motions such as bouncing, rolling, and tumbling, is modeled by choosing 100 random points in the vicinity, of each landing position as a take-off position for the subsequent hop, that is, we generate 100 hopping trajectories. Then, from these trajectories, we select the one that corresponds to the farthest landing point (that is, we select the worst case) and then proceed again with the generation of more 100 random trajectories around the selected landing point. The distribution of these random points is Gaussian with variance $(\sigma_x^2, \sigma_y^2, \sigma_z^2) = (1, 1, 0.1)$ m^2. Corresponding uncertain trajectory and respective distance errors at each landing position are plotted by the blue dotted trajectories and as the box-plot in Figs. 6a and 6b, respectively, which show the propagation of positioning error that obviously increases in the absence of replanning because it is continuously integrating the error. The proposed approach can be used with a localization routine to replan a feasible path from the drifted position to goal. A complementary solution is to select small friction cones in the direction of $Z-$axis such that it increases the vertical rebound component of the robot as compare to horizontal component which result in lower drifting error in case of bouncing or tumbling.

VI. CONCLUSION

We have discussed the CHTA problem for non-uniform polygonal chains and proposed an algorithm to determine the traversability of constrained hopping path on the polygon chains. We imposed the line and terrain avoidance constraints which result in a feasible range of take-off angles. A closed-form expression to compute a minimum number of hops for a feasible optimal take-off angle is derived and used as hopping cost between any two points on the chain. Using discretization and a graph search approach, we solve the CHTA problem for a given source and goal on the chain and generated a corresponding feasible hopping path with minimum number of hops and least take-off speed. Performance of the proposed approach on different resolutions (discretization step size) and its applicability to path planning scenarios are discussed with illustrative examples. The proposed algorithms are computational efficient, which has practical significance in hopping robot applications.

APPENDIX

BASICS OF POINT-TO-POINT BALLISTIC HOP

Without loss of generality, consider take-off time $t_t = 0$ and integrate (1)-(2) which gives the velocity and position components of the hopping trajectory as

$$\dot{x}(t) = v_t \cos(\theta_t) \tag{31}$$

$$\dot{z}(t) = v_t \sin(\theta_t) - gt \tag{32}$$

$$x(t) = x_t + v_t \cos(\theta_t)t \tag{33}$$

$$z(t) = z_t + v_t \sin(\theta_t)t - \frac{gt^2}{2} \tag{34}$$

For sake of simplicity, we drop the time notation (e.g. $x(t)$ is written as x), hereinafter. From (33) and (34), the explicit equation of the ballistic hop can be deduced as

$$z(x) = z_t + \tan(\theta_t)(x - x_t) - \frac{g(x - x_t)^2}{2v_t^2 \cos^2(\theta_t)}$$

Next, using (33) and (34), the line-of-sight (LOS) distance and angle between take-off and landing points can be obtained as

$$R = \sqrt{(x_t - x_l)^2 + (z_t - z_l)^2} \quad \text{and}$$
$$\beta = \tan^{-1} \frac{z_l - z_t}{x_l - x_t},$$

respectively. The position coordinates of the landing points in terms of LOS distance and angle can be given as

$$x_l = x_t + R\cos\beta \tag{35}$$

$$z_l = z_t + R\sin\beta \tag{36}$$

Useful expressions for the point-to-point ballistic hop are given as follows:

1) **Traveling time:** From (4a) and (35), at the landing point

$$x_t + R\cos\beta = x_t + v_t\cos(\theta_t)(t_l - t_t)$$
$$\implies T := t_l - t_t = \frac{R\cos\beta}{v_t\cos(\theta_t)} \tag{37}$$

where T represent the traveling time to reach (x_l, z_l) from (x_t, z_t).

2) **Take-off speed:** The take-off speed required to reach the landing point can be obtained by using (36) and (8) in (34) as follows

$$v_t \sin(\theta_t)T - \frac{gT^2}{2} = R\sin(\beta)$$
$$v_t = \sqrt{\frac{Rg\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)}} \quad (38)$$

3) **Landing speed and angle:** Using (31) and (32), the landing speed at the landing point (x_t, z_t) can be computed as

$$\begin{aligned} v_l &= \sqrt{\dot{x}^2(T) + \dot{z}^2(T)} \\ \implies v_l &= \sqrt{v_t^2 + (gT)^2 - 2gv_t\sin(\theta_t)T} \end{aligned} \quad (39)$$

Substituting T and v_t from (37) and (38), respectively in (39), yields

$$v_l = \sqrt{\frac{Rg\cos^2(\beta)}{2\cos(\theta_t)\sin(\theta_t - \beta)} - 2Rg\sin(\beta)}$$

From (11) one can note that for a given distance R, and fixed angles β and θ_t, the pair of initial and impact speeds are completely defined. Similarly, landing angle θ_l can be obtained as follows:

$$\begin{aligned} \theta_l &= \tan^{-1}\left(\frac{\dot{z}(T)}{\dot{x}(T)}\right) \\ \implies \theta_l &= \tan^{-1}\left(2\tan(\beta) - \tan(\theta_t)\right) \end{aligned} \quad (40)$$

which is a unique relationship between θ_t and θ_l for a fixed β.

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REFERENCES

[1] J. K. Yim and R. S. Fearing, "Precision jumping limits from flight-phase control in salto-1P," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Madrid, Spain, Oct. 2018, pp. 2229–2236.
[2] S. Fortune and G. Wilfong, "Planning constrained motion," *Ann. Math. Artif. Intell.*, vol. 3, no. 1, pp. 21–82, Mar. 1991.
[3] H.-K. Ahn, O. Cheong, J. Matoušek, and A. Vigneron, "Reachability by paths of bounded curvature in a convex polygon," *Comput. Geometry*, vol. 45, nos. 1–2, pp. 21–32, Jan. 2012.
[4] R. V. Cowlagi and P. Tsiotras, "Curvature-bounded traversability analysis in motion planning for mobile robots," *IEEE Trans. Robot.*, vol. 30, no. 4, pp. 1011–1019, Aug. 2014.
[5] S. Upadhyay and A. Ratnoo, "On existence and synthesis of smooth four parameter logistic paths inside annular passages," *IEEE Robot. Autom. Lett.*, vol. 3, no. 4, pp. 4375–4382, Oct. 2018.
[6] B. Hockman and M. Pavone, "Traversability of hopping rovers on small solar system bodies," in *Proc. Int. Symp. Artif. Intell. Robot. Automat. Space*, Madrid, Spain, Jul. 2018, pp. 1–8.
[7] J. Burdick and P. Fiorini, "Minimalist jumping robots for celestial exploration," *Int. J. Robot. Res.*, vol. 22, nos. 7–8, pp. 653–674, Jul. 2016.
[8] T. Yoshimitsu, I. Nakatani, and T. Kubota, "New mobility system for small planetary body exploration," in *Proc. IEEE Int. Conf. Robot. Automat.*, Detroit, MI, USA, vol. 2, May 1999, pp. 1404–1409.
[9] A. Sayyad, B. Seth, and P. Seshu, "Single-legged hopping robotics research—A review," *Robotica*, vol. 25, no. 5, pp. 587–613, Sep. 2007.
[10] L. Zhang, H. Chen, and Y. Yang, "Review on mechanism and modeling of jumping robot," in *Proc. IEEE Int. Conf. Cyber Technol. Automat., Control, Intell. Syst. (CYBER)*, Chengdu, China, Jun. 2016, pp. 255–260.
[11] Y. Zhang, L. Zhang, W. Wang, Y. Li, and Q. Zhang, "Design and implementation of a two-wheel and hopping robot with a linkage mechanism," *IEEE Access*, vol. 6, pp. 42422–42430, 2018.
[12] M. Ushijima, Y. Kunii, T. Maeda, T. Yoshimitsu, and M. Otsuki, "Path planning with risk consideration on hopping mobility," in *Proc. IEEE/SICE Int. Symp. Syst. Integr. (SII)*, Taipei, Taiwan, Dec. 2017, pp. 692–697.
[13] M. Campana and J.-P. Laumond, "Ballistic motion planning," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Daejeon, South Korea, Oct. 2016, pp. 1410–1416.
[14] S. Levine, Y. Lee, V. Koltun, and Z. Popović, "Space-time planning with parameterized locomotion controllers," *ACM Trans. Graph.*, vol. 30, no. 3, pp. 1–11, May 2011.
[15] K. Kim, L.-H. Chen, B. Cera, M. Daly, E. Zhu, J. Despois, A. K. Agogino, V. Sunspiral, and A. M. Agogino, "Hopping and rolling locomotion with spherical tensegrity robots," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Daejeon, South Korea, Oct. 2016, pp. 4369–4376.
[16] S. Upadhyay and A. P. Aguiar, "Local trajectory generation for hopping robots exploring celestial bodies," in *Proc. AIAA SciTech Forum, Guid. Navigat., Control Conf.*, San Diago, CA, USA, Jan. 2019, Art. no. AIAA 2019-1913.
[17] J. Barraquand and J-C. Latombe, "Nonholonomic multibody mobile robots: Controllability and motion planning in the presence of obstacles," *Algorithmica*, vol. 10, nos. 2–4, pp. 121–155, Oct. 1993.
[18] J.-C. Latombe, *Robot Motion Planning*. Boston, MA, USA: Academic, 1991.
[19] *Mars Trek*. Accessed: Sep. 28, 2018. [Online]. Available: https://trek.nasa.gov/mars/

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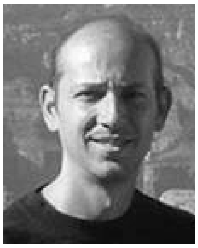

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